PERFORMANCE ANALYSIS OF TRUNCATED MULTI-CHANNEL QUEUE

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Abstract: *The paper is aimed to focus on the performance analysis of a truncated multi-channel $M/M/C/K$ queuing system to be used in the machine interference models arising out of industrial and computer manufacturing etc. A total cost function is subjected to the optimization in view of arrival and service parameters by using a computing algorithm of numerical methods. The optimal values of various performance measures of the system such as optimal number of machine-customers, optimal waiting time for repair, optimal traffic intensity are evaluated by using hypothetical data-input corresponding to the optimal total cost of the system. Results are tabled and also presented graphically to better gain the insight into applications of derived performance measures of the system in different working conditions.*

Keywords: *System of non-linear equations; total optimal cost; arrival rate; service rate.*

1. INTRODUCTION

Performance analysis of the queuing models occupies a prominent place in the research of queuing theory, a significant aspect of the optimization theory. Optimization techniques are widely used in the areas of production, manufacturing, and planning including the communication systems to effectively assess the performance of the systems. It has drawn the attention of the researchers seriously engaged in this area of research. Also, to date it reveals that no sufficient work has been done with regard to computation of optimal performance measures of the queuing system by optimizing both the parameters, arrival and service rates, in general and truncated multi-channel queuing system in particular.

Nowadays, a trend has been redirected and shifted to investigate more realistic performance measures of the system as compared to general theoretical approach that embodies hardly a bit of application. Some of the relevant researches are in sequel here. Chakravarthy et al. [2] considered a multi-server queuing model in which customers arrive according to Markovian arrival process (MAP). They have performed steady-state analysis of the model using direct truncation and matrix-geometric approximation. Efficient algorithms for computing various steady-state performance measures and illustrative numerical examples have also been presented. Artalejo and Gomez-Corral [1] shown that the limiting distribution of the system state can still be reduced to a Fredholm integral equation. They solved such an equation numerically by introducing an auxiliary truncated system which can easily be evaluated with the help of regenerative approach.

Tirtiroglu [15] has presented an entropy based uncertainty metric for measuring the operating performance of $M/M/1$ and $M/M/1/K$ models. The author considered a connection between entropy and the uncertainty in queuing. El-Taha and Maddah [4] have considered a multi-server first come, first serve (FCFS) queuing model where servers are arranged in two stations in series. They have shown that their scheme provides better system performance than the standard parallel multi-server model in the sense of reducing the mean delay per customer in heavy traffic system. Naor [10] discussed the pricing problem by giving quantitative arguments based on an $M/M/1/K$ queuing model. In this work, he has shown the necessity of limiting the arrivals to a queuing system by a toll to achieve the social optimality. Knudsen [6] has extended Naor's study to a multi-server queuing system. Wang et al. [3] have considered an unloader queuing model in which N identical trailers are unloaded by one or more unloaders and developed a cost model to determine the optimal number of trailers.

Shawky and El-Paoumy [13] treated the truncated multi-channel queue $H_k/M/c/N$ with both balking and reneging concepts. They derived steady-state probabilities of the model together with some measures of effectiveness where these measures were analytically deduced. Taha [14] discussed the two conflicting costs viz. cost of offering the service and the cost of delay in offering the service and established the cost model for the system. He also derived formulas to evaluate the performance measures of various queuing systems. Shawky [12] analyzed the machine interference model $M/M/C/K/N$ with balking, reneging, and spares. He has presented the steady-state probabilities and expected number of customers in the system for four different cases. He also considered the truncated multi-channel queue $M/M/C/K$ as one of the cases under consideration. Gross and Harris [5] have discussed the $M/M/1/K$ queuing system with truncation. They derived steady-state probabilities for the system and obtained formulas for various performance measures of the system. They also discussed the performance measures of the multi-channel queue $M/M/C/K$ with truncation.

Mishra and Mishra [7] discussed the cost analysis of the machine interference model $M/M/C/K/N$. Here, they constructed a cost function in order to determine the total optimal cost of the system. A fast converging N-R method has been used to solve the non-linear function involving service rate and hyper geometric functions including other parameters. They optimized the total cost function with respect to single parameter, service rate μ. Morse [9] has solved the queuing system with hyper-Poisson arrivals and a single exponential channel without balking or reneging. Neuts and Lucantoni [11] studied a queue with N servers who may breakdown and repair at a facility which has c

repair crews. They discussed the stationary distributions of various waiting times. Mishra [8] has made an attempt to compute the total optimal cost of interdependent queuing system with controllable arrival rates as an important performance measure of the system.

. In this paper, we define a total cost function for the system and apply optimization with respect to both the parameters arrival rate and service rate simultaneously. For computing the total optimal cost of the system and other performance measures, like optimal expected number of customers in the system, optimal waiting time in the system, and optimal traffic intensity of the system, a computing algorithm has been developed. Finally, numerical demonstrations in the form of tables and graphs are added to gain a significant insight into the problem. Various observations are drawn to realize the problem closely related to real life situations.

2. PERFORMANCE ANALYSIS OF THE MODEL

For the truncated multi-channel queue $M/M/C/K$ (Poisson arrival and exponential service), the system probabilities are as given by Shawky [12],

$$P_n = \begin{cases} \dfrac{\rho^n}{n!} P_0 \,;\ 0 \le n < C \\ \dfrac{\rho^n}{C! C^{n-C}} P_0 \,;\ C \le n \le K \end{cases}$$

$$P_0^{-1} = \sum_{n=0}^{C-1} \frac{\rho^n}{n!} + \frac{\rho^C}{(C-1)!} \cdot \frac{1-(\rho/C)^{K-C+1}}{(C-\rho)} \ ; \quad \rho \ne C$$

$$P_0^{-1} = \sum_{n=0}^{C-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^C}{(C-1)!} \cdot \frac{1-(\lambda/C\mu)^{K-C+1}}{\left(C-\dfrac{\lambda}{\mu}\right)} ; \ \frac{\lambda}{\mu} \ne C \tag{1}$$

where P_0 is the empty system probability, C is the number of servers, K is the capacity of the system, λ is arrival rate, μ is service rate, and ρ is the traffic intensity.

Expected number of customers in the system L_s is given by (Shawky [12]),

$$L_s = P_0 \left[\sum_{n=1}^{C-1} \frac{\rho^n}{(n-1)!} + \frac{\rho C^{C+1}}{C!(C-\rho)^2} \left\{ C(\rho/C)^{C-1} - (C-1)(\rho/C)^C - (K+1)(\rho/C)^K + K(\rho/C)^{K+1} \right\} \right]$$

$$\text{Or, } L_s = P_0 \left[\sum_{n=1}^{C-1} \frac{\rho^n}{(n-1)!} + \frac{\rho^C}{(C-1)!} \cdot \frac{\left\{C^2-(C-1)\rho\right\}}{(C-\rho)^2} + \frac{C^{C-K} \rho^{K+1}}{C!} \cdot \frac{\{K\rho - C(K+1)\}}{(C-\rho)^2} \right]$$

$$\text{Or, } L_s = P_0 \left[\sum_{n=1}^{C-1} \frac{(\lambda/\mu)^n}{(n-1)!} + \frac{(\lambda/\mu)^C}{(C-1)!} \cdot \frac{\left\{C^2-(C-1)(\lambda/\mu)\right\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2} + \frac{C^{C-K} (\lambda/\mu)^{K+1}}{C!} \cdot \frac{\{K(\lambda/\mu) - C(K+1)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2} \right]$$

Let $P_0^{-1} = f$, and

$$M = \sum_{n=1}^{C-1} \frac{1}{(n-1)!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{(C-1)!\left(C-\dfrac{\lambda}{\mu}\right)^2} \times \left[C^2 \left(\frac{\lambda}{\mu}\right)^C - (C-1)\left(\frac{\lambda}{\mu}\right)^{C+1} - (K+1)C^{C-K}\left(\frac{\lambda}{\mu}\right)^{K+1} + K\,C^{C-K-1}\left(\frac{\lambda}{\mu}\right)^{K+2} \right] \tag{2}$$

Therefore,

$$L_s = M\,P_0 = \frac{M}{f}$$

Differentiating (2) partially with respect to λ, we get

$$\frac{\partial M}{\partial \lambda} = \sum_{n=1}^{C-1} \frac{n}{(n-1)!} \frac{\lambda^{n-1}}{\mu^n} + \frac{C}{(C-1)!} \frac{\lambda^{C-1}}{\mu^C} \frac{\left\{C^2-(C-1)(\lambda/\mu)\right\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2} + \frac{1}{(C-1)!} \frac{\lambda^C}{\mu^{C+1}} \frac{\{C(C+1)-(C-1)(\lambda/\mu)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^3} + \frac{(K+1)C^{C-K}}{C!} \frac{\lambda^K}{\mu^{K+1}} \frac{\{K(\lambda/\mu)-C(K+1)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2} + \frac{C^{C-K}}{C!} \frac{\lambda^{K+1}}{\mu^{K+2}} \frac{\{K(\lambda/\mu)-C(K+2)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^3} \tag{3}$$

Differentiating (2) partially with respect to μ, we get

$$\frac{\partial M}{\partial \mu} = -\sum_{n=1}^{C-1} \frac{n}{(n-1)!} \frac{\lambda^n}{\mu^{n+1}} - \frac{C}{(C-1)!} \frac{\lambda^C}{\mu^{C+1}} \frac{\left\{C^2-(C-1)(\lambda/\mu)\right\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2} - \frac{1}{(C-1)!} \frac{\lambda^{C+1}}{\mu^{C+2}} \frac{\{C(C+1)-(C-1)(\lambda/\mu)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^3} - \frac{(K+1)C^{C-K}}{C!} \frac{\lambda^{K+1}}{\mu^{K+2}} \frac{\{K(\lambda/\mu)-C(K+1)\}}{\left(C-\dfrac{\lambda}{\mu}\right)^2}$$

$$-\frac{C^{C-K}}{C!}\frac{\lambda^{K+2}}{\mu^{K+3}}\frac{\{K(\lambda/\mu)-C(K+2)\}}{\left(C-\frac{\lambda}{\mu}\right)^3} \tag{4}$$

Differentiating (1) partially with respect to λ and μ respectively, we have

$$\frac{\partial f}{\partial \lambda}=\sum_{n=1}^{C-1}\frac{1}{(n-1)!}\frac{\lambda^{n-1}}{\mu^n}+\frac{1}{(C-1)!\left(C-\frac{\lambda}{\mu}\right)^2}\times\left[C^2\frac{\lambda^{C-1}}{\mu^C}-(C-1)\frac{\lambda^C}{\mu^{C+1}}-(K+1)C^{C-K}\frac{\lambda^K}{\mu^{K+1}}+K\,C^{C-K-1}\frac{\lambda^{K+1}}{\mu^{K+2}}\right] \tag{5}$$

$$\frac{\partial f}{\partial \mu}=-\sum_{n=1}^{C-1}\frac{1}{(n-1)!}\frac{\lambda^{n}}{\mu^{n+1}}-\frac{1}{(C-1)!\left(C-\frac{\lambda}{\mu}\right)^2}\times\left[C^2\frac{\lambda^{C}}{\mu^{C+1}}-(C-1)\frac{\lambda^{C+1}}{\mu^{C+2}}-(K+1)C^{C-K}\frac{\lambda^{K+1}}{\mu^{K+2}}+K\,C^{C-K-1}\frac{\lambda^{K+2}}{\mu^{K+3}}\right] \tag{6}$$

From (5) and (2), we observe that

$$M=\lambda\frac{\partial f}{\partial\lambda} \tag{7}$$

From (6) and (2), we observe that

$$M=-\mu\frac{\partial f}{\partial\mu} \tag{8}$$

From (5) and (6), we observe that

$$\frac{\partial f}{\partial\mu}+\frac{\lambda}{\mu}\frac{\partial f}{\partial\lambda}=0$$

From (3) and (4), we observe that

$$\frac{\partial M}{\partial\mu}+\frac{\lambda}{\mu}\frac{\partial M}{\partial\lambda}=0 \tag{9}$$

The total cost function $TC=C_1(m+C)\mu+C_2\frac{M}{f}$ is a function in two variables λ and μ. Differentiating this equation partially with respect to λ and μ respectively, we get:

$$\frac{\partial(TC)}{\partial\lambda}=C_2\frac{f\frac{\partial M}{\partial\lambda}-M\frac{\partial f}{\partial\lambda}}{f^2} \tag{10}$$

$$\frac{\partial(TC)}{\partial\mu}=C_1(m+C)+C_2\frac{f\frac{\partial M}{\partial\mu}-M\frac{\partial f}{\partial\mu}}{f^2} \tag{11}$$

For critical point $(\bar\lambda,\bar\mu)$, we must have $\frac{\partial(TC)}{\partial\lambda}=\frac{\partial(TC)}{\partial\mu}=0$. Therefore from (10) and (11), we have

$$f\frac{\partial M}{\partial\lambda}-M\frac{\partial f}{\partial\lambda}=0 \tag{12}$$

$$C_1(m+C)f^2+C_2\left[f\frac{\partial M}{\partial\mu}-M\frac{\partial f}{\partial\mu}\right]=0 \tag{13}$$

Using (7) and (8) in (12) and (13), we get

$$f\frac{\partial M}{\partial\lambda}-\frac{M^2}{\lambda}=0,\ C_1(m+C)f^2+C_2\left[f\frac{\partial M}{\partial\mu}+\frac{M^2}{\mu}\right]=0$$

Now, let $\phi\equiv\phi(\lambda,\mu)=f\frac{\partial M}{\partial\lambda}-\frac{M^2}{\lambda}$ and

$\psi\equiv\psi(\lambda,\mu)=C_1(m+C)f^2+C_2\left[f\frac{\partial M}{\partial\mu}+\frac{M^2}{\mu}\right]$, then

$$\phi(\lambda,\mu)=0 \tag{14}$$

$$\text{and } \psi(\lambda,\mu)=0 \tag{15}$$

constitute a set of two non-linear equations in two variables λ and μ. We shall solve these two equations by applying fast converging Newton-Raphson's method to obtain critical point $(\bar\lambda,\bar\mu)$.

If (λ_i,μ_i) is the initial guess for equations (14) and (15) then we have,

$$\lambda_{i+1}-\lambda_i=-\frac{\phi_i(\partial\psi_i/\partial\mu)-\psi_i(\partial\phi_i/\partial\mu)}{\delta}$$

$$\mu_{i+1}-\mu_i=-\frac{\psi_i(\partial\phi_i/\partial\lambda)-\phi_i(\partial\psi_i/\partial\lambda)}{\delta}$$, where

$$\delta=\frac{\partial\phi_i}{\partial\lambda}\cdot\frac{\partial\psi_i}{\partial\mu}-\frac{\partial\phi_i}{\partial\mu}\cdot\frac{\partial\psi_i}{\partial\lambda}$$

By partial differentiation, we have

$$\frac{\partial\phi}{\partial\lambda}=f\frac{\partial^2 M}{\partial\lambda^2}-\frac{M}{\lambda}\frac{\partial M}{\partial\lambda}+\left(\frac{M}{\lambda}\right)^2 \tag{16}$$

$$\frac{\partial\phi}{\partial\mu}=-f\left(\frac{\mu}{\lambda}\right)\frac{\partial^2 M}{\partial\mu^2}-\frac{\partial M}{\partial\mu}\frac{(M+f)}{\lambda} \tag{17}$$

$$\frac{\partial\psi}{\partial\lambda}=-C_2\,f\left(\frac{\lambda}{\mu}\right)\frac{\partial^2 M}{\partial\lambda^2}+\frac{C_2}{\mu}\frac{\partial M}{\partial\lambda}\{M-f\}+2C_1(m+C)f\frac{M}{\lambda} \tag{18}$$

$$\frac{\partial\psi}{\partial\mu}=C_2\,f\frac{\partial^2 M}{\partial\mu^2}+C_2\left(\frac{M}{\mu}\right)\frac{\partial M}{\partial\mu}-\left\{2C_1(m+C)\,f\left(\frac{M}{\mu}\right)+C_2\frac{M^2}{\mu^2}\right\} \tag{19}$$

From (16), (17), (18), and (19), it is clear that we have to find $\frac{\partial^2 M}{\partial\lambda^2}$ and $\frac{\partial^2 M}{\partial\mu^2}$.

$$\frac{\partial^2 M}{\partial\lambda^2}=\sum_{n=2}^{C-1}\frac{n}{(n-2)!}\frac{\lambda^{n-2}}{\mu^n}+\frac{C}{(C-2)!}\frac{\lambda^{C-2}}{\mu^C}\frac{\{C^2-(C-1)(\lambda/\mu)\}}{\left(C-\frac{\lambda}{\mu}\right)^2}$$

$$+\frac{2C}{(C-1)!}\frac{\lambda^{C-1}}{\mu^{C+1}}\frac{\{C(C+1)-(C-1)(\lambda/\mu)\}}{\left(C-\frac{\lambda}{\mu}\right)^3}$$

$$+\frac{2}{(C-1)!}\frac{\lambda^{C}}{\mu^{C+2}}\frac{\{C(C+2)-(C-1)(\lambda/\mu)\}}{\left(C-\frac{\lambda}{\mu}\right)^4}$$

$$+\frac{K(K+1)C^{C-K}}{C!}\frac{\lambda^{K-1}}{\mu^{K+1}}\frac{\{K(\lambda/\mu)-C(K+1)\}}{\left(C-\frac{\lambda}{\mu}\right)^2}$$

$$+\frac{2(K+1)C^{C-K}}{C!}\frac{\lambda^{K}}{\mu^{K+2}}\frac{\{K(\lambda/\mu)-C(K+2)\}}{\left(C-\frac{\lambda}{\mu}\right)^3}$$

$$+\frac{C^{C-K}}{C!}\frac{\lambda^{K+1}}{\mu^{K+3}}\frac{2\{K(\lambda/\mu)-C(K+3)\}}{\left(C-\frac{\lambda}{\mu}\right)^4}$$

$$\frac{\partial^2 M}{\partial\mu^2}=\sum_{n=1}^{C-1}\frac{n(n+1)}{(n-1)!}\frac{\lambda^n}{\mu^{n+2}}$$

$$+\frac{C(C+1)}{(C-1)!}\frac{\lambda^{C}}{\mu^{C+2}}\left\{\frac{C^2-(C-1)(\lambda/\mu)}{\left(C-\frac{\lambda}{\mu}\right)^2}\right\}$$

$$+\frac{2(C+1)}{(C-1)!}\frac{\lambda^{C+1}}{\mu^{C+3}}\left\{\frac{C(C+1)-(C-1)(\lambda/\mu)}{\left(C-\frac{\lambda}{\mu}\right)^3}\right\}$$

$$+\frac{2}{(C-1)!}\frac{\lambda^{C+2}}{\mu^{C+4}}\frac{\{C(C+2)-(C-1)(\lambda/\mu)\}}{\left(C-\frac{\lambda}{\mu}\right)^4}$$

$$+\frac{(K+2)(K+1)C^{C-K}}{C!}\frac{\lambda^{K+1}}{\mu^{K+3}}\left\{\frac{K(\lambda/\mu)-C(K+1)}{\left(C-\frac{\lambda}{\mu}\right)^2}\right\}$$

$$+\frac{2(K+2)C^{C-K}}{C!}\frac{\lambda^{K+2}}{\mu^{K+4}}\frac{\{K(\lambda/\mu)-C(K+2)\}}{\left(C-\frac{\lambda}{\mu}\right)^3}$$

$$+\frac{2\,C^{C-K}}{C!}\frac{\lambda^{K+3}}{\mu^{K+5}}\frac{\{K(\lambda/\mu)-C(K+3)\}}{\left(C-\frac{\lambda}{\mu}\right)^4}$$

The total expected cost of the system TC will be optimal at $(\bar{\lambda},\bar{\mu})$ if, $\frac{\partial^2 TC}{\partial\lambda^2}\cdot\frac{\partial^2 TC}{\partial\mu^2}-\frac{\partial^2 TC}{\partial\lambda\,\partial\mu}\cdot\frac{\partial^2 TC}{\partial\mu\,\partial\lambda}>0$ and $\frac{\partial^2 TC}{\partial\lambda^2}>0$ where,

$$\frac{\partial^2 TC}{\partial\lambda^2}=\frac{C_2}{f}\left[\frac{\partial^2 M}{\partial\lambda^2}-3\frac{M}{f\lambda}\frac{\partial M}{\partial\lambda}+\left(\frac{M}{f\lambda}\right)^2(f+2M)\right],$$

$$\frac{\partial^2 TC}{\partial\mu^2}=\frac{C_2}{f}\left[\frac{\partial^2 M}{\partial\mu^2}+3\frac{M}{f\mu}\frac{\partial M}{\partial\mu}-\left(\frac{M}{\mu f}\right)^2(f-2M)\right],$$

$$\frac{\partial^2 TC}{\partial\lambda\,\partial\mu}=\frac{C_2}{f\mu}\left[-\frac{\partial M}{\partial\lambda}-\lambda\frac{\partial^2 M}{\partial\lambda^2}+\frac{3M}{f}\frac{\partial M}{\partial\lambda}-\frac{2M^3}{\lambda f^2}\right], \text{ and}$$

$$\frac{\partial^2 TC}{\partial\mu\,\partial\lambda}=-\frac{C_2}{f\lambda}\left[\frac{\partial M}{\partial\mu}+\mu\frac{\partial^2 M}{\partial\mu^2}+\frac{3M}{f}\frac{\partial M}{\partial\mu}+\frac{2M^3}{\mu f^2}\right]$$

The above conditions are sufficient for the total expected cost of the system TC to be optimal at $(\bar{\lambda},\bar{\mu})$. We find the various performance measures of the system which are optimal expected number of customers in the system $\overline{L_s}$, optimal waiting time in the system $\overline{W_s}$, and optimal traffic intensity $\bar{\rho}$.

3. PUTING ALGORITHM

The following computing algorithm is developed to find out the optimal arrival and service rates, total optimal cost, and other performance measures of the system.

Step 1: begin
Step 2: input all the parameters
Step 3: input initial guess for arrival and service rates
Step 4: compute all the derivatives
Step 5: iterating arrival and service rates
Step 6: compute optimal arrival and service rates
Step 7: compute optimal performance measures
Step 8: compute total optimal cost
Step 9: data output
Step 10: end

4. SENSITIVITY ANALYSIS OF THE MODEL

The aim of the sensitivity analysis is to demonstrate the variability of the model based on the simulations or the hypothetical data-input. In this chapter, we prefer the hypothetical data-input to run the search program of the system. We wrote a program in C++ to apply a two-variable version of N-R method to compute the optimal arrival and service rates and consequently the total optimal cost of the system, optimal expected number of customers in the system, optimal waiting time in the system, and optimal traffic intensity of the system are also computed. In sensitivity analysis, variation effect of parameters on the total optimal cost and other performance measures is presented in the form of graphs and tables.

Table no 1: Service Cost vs. Total Optimal Cost and various Performance Measures

$C = 4,\ K = 20,\ m = 3,\ C_2 = 2.50$

(C_1)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\overline{TC})$	$\bar{L}_s$	$\bar{W}_s$	$\bar{\rho}$
3.50	2.62	4.79	121.92	5.4	2.06	0.55
4.00	2.66	4.80	138.91	6.7	2.52	0.55
4.50	2.69	4.80	155.66	8.1	3.01	0.56
5.50	2.73	4.81	189.59	9.3	3.41	0.57
6.50	2.77	4.82	223.66	11.4	4.12	0.57
7.50	2.79	4.83	257.90	12.2	4.37	0.58
8.50	2.81	4.83	291.68	13.0	4.63	0.58
9.50	2.82	4.83	325.47	15.6	5.53	0.58
10.50	2.83	4.84	360.01	17.7	6.25	0.58

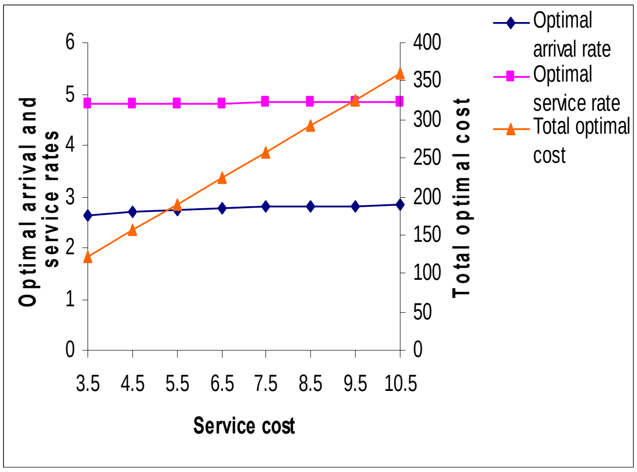

Fig.1.1: Service Cost vs. Total Optimal Cost

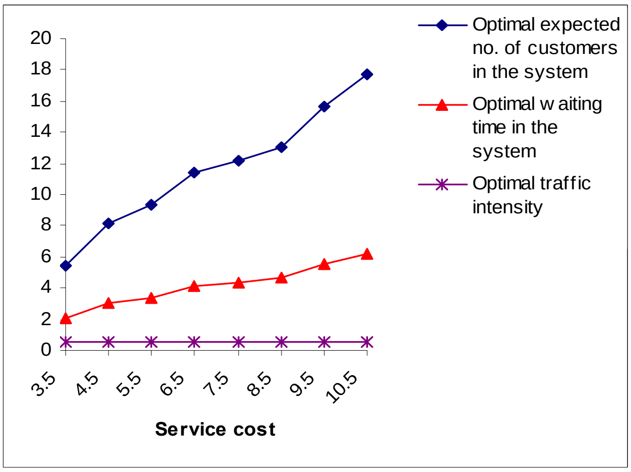

Fig.1.2: Service Cost vs. various Performance Measures

Table no 2: Waiting Cost vs. Total Optimal Cost and various Performance Measures

$C = 4,\ K = 20,\ m = 3,\ C_1 = 10.50$

(C_2)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\overline{TC})$	$\bar{L}_s$	$\bar{W}_s$	$\bar{\rho}$
2.50	2.83	4.84	360.01	17.7	6.25	0.58
3.00	2.81	4.83	360.16	17.2	6.12	0.58
4.00	2.77	4.82	361.22	16.5	5.96	0.57
5.00	2.73	4.81	362.34	15.8	5.79	0.57
6.00	2.68	4.80	363.54	15.0	5.60	0.56
7.00	2.64	4.79	364.76	14.2	5.38	0.55
8.00	2.60	4.78	366.03	13.5	5.19	0.54
9.00	2.55	4.77	367.42	12.7	4.98	0.53

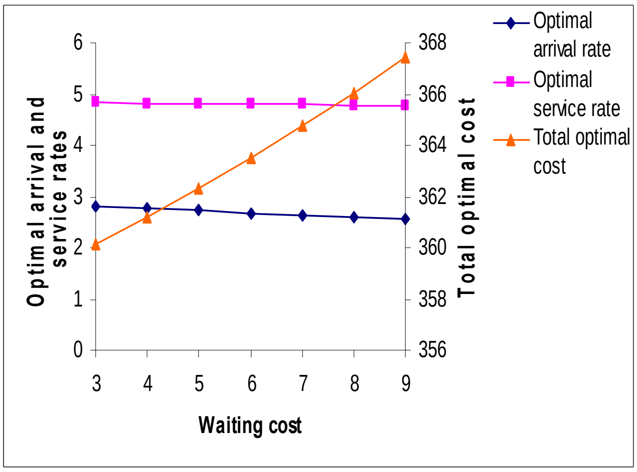

Fig.2.1: Waiting Cost vs. Total Optimal Cost

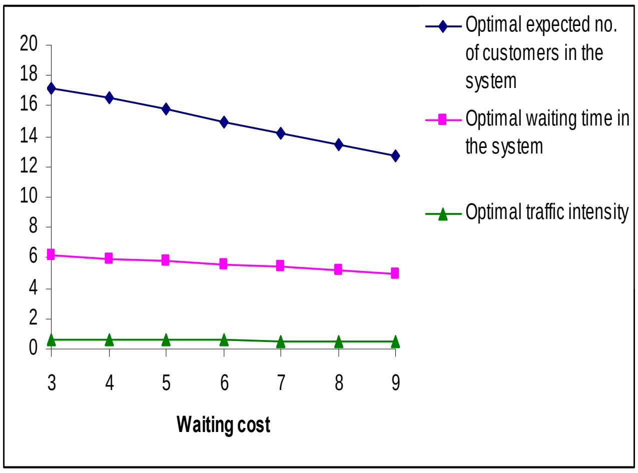

Fig.2.2: Waiting Cost vs. various Performance Measures

Table- 3: No. of Servers vs. Total Optimal Cost and various Performance Measures

$C_1 = 3.50,\ C_2 = 2.50,\ m = 3,\ K = 20$

(C)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\overline{TC})$	$\bar{L}_s$	$\bar{W}_s$	$\bar{\rho}$
3	1.37	1.77	40.33	8.4	6.13	0.77
4	2.62	4.79	121.92	5.3	2.02	0.55
5	2.49	5.38	156.04	5.8	2.33	0.46
6	1.72	4.69	154.55	4.9	2.85	0.37
7	1.11	3.97	147.89	3.6	3.24	0.30

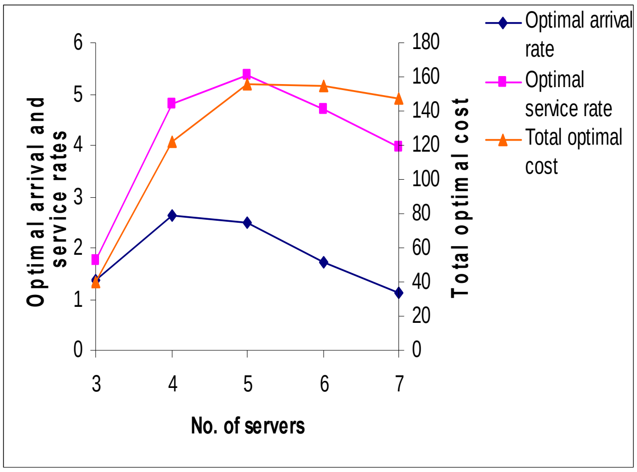

Fig.3.1: No. of Servers vs. Total Optimal Cost

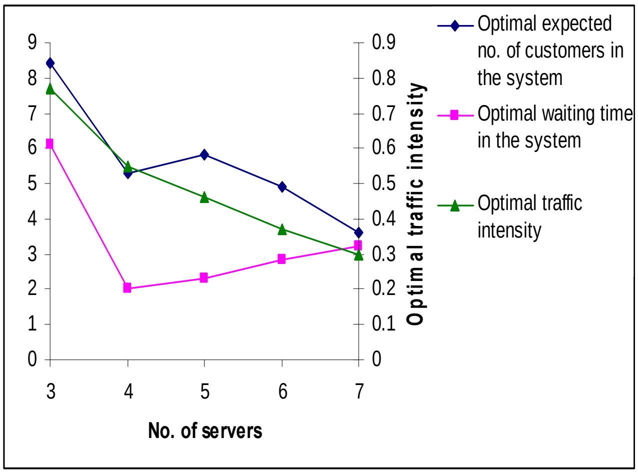

Fig.3.2: No. of Servers vs. various Performance Measures

Table- 4: No. of Customers in the System (m) vs. Total Optimal Cost and various Performance Measures

$C = 7,\ K = 20,\ C_1 = 3.50,\ C_2 = 2.50,\ m < C$

(m)	$(\bar{\lambda})$	$(\bar{\mu})$	$(\overline{TC})$	$\bar{L}_s$	$\bar{W}_s$	$\bar{\rho}$
3	1.11	3.97	147.89	3.6	3.24	0.28
4	1.13	3.98	162.04	3.8	3.36	0.28
5	1.14	3.99	176.33	3.8	3.33	0.29
6	1.77	4.72	221.43	4.4	2.49	0.38

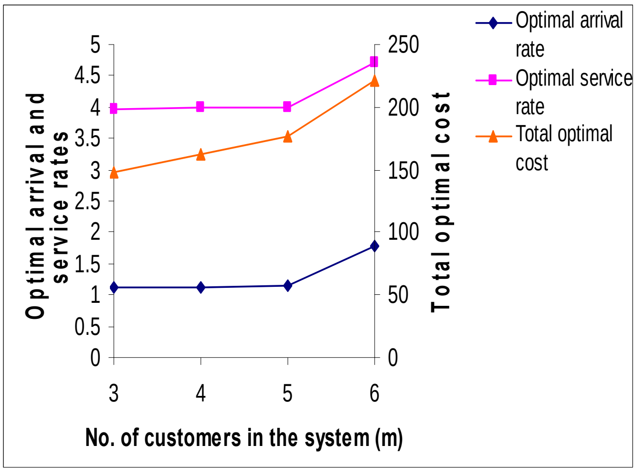

Fig.4.1: No. of Customers in the System (m) vs. Total Optimal Cost

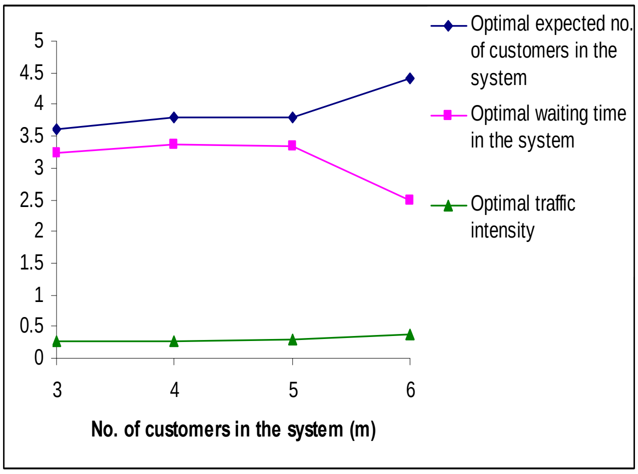

Fig.4.2: No. of Customers in the System (m) vs. various Performance Measures

Table no 5: Capacity of the System vs. Total Optimal Cost and various Performance Measures

$C = 4,\ m = 3,\ C_1 = 3.50,\ C_2 = 2.50$

(K)	$(\bar{\lambda})$	$(\bar{\mu})$	$\overline{TC}$	$\bar{L}_s$	$\bar{W}_s$	$\bar{\rho}$
20	2.62	4.79	121.92	5.3	2.02	0.55
22	2.59	4.76	121.21	5.8	2.24	0.54
24	2.57	4.74	120.74	6.5	2.53	0.54
26	2.56	4.73	120.50	6.9	2.70	0.54
28	2.55	4.72	120.26	7.3	2.86	0.54
30	2.55	4.72	120.26	8.1	3.18	0.54
32	2.55	4.72	120.26	8.8	3.45	0.54
34	2.55	4.71	120.00	9.4	3.69	0.54
36	2.55	4.71	120.00	10.1	3.96	0.54
38	2.54	4.71	120.03	10.7	4.21	0.54
40	2.54	4.71	120.03	11.0	4.33	0.54

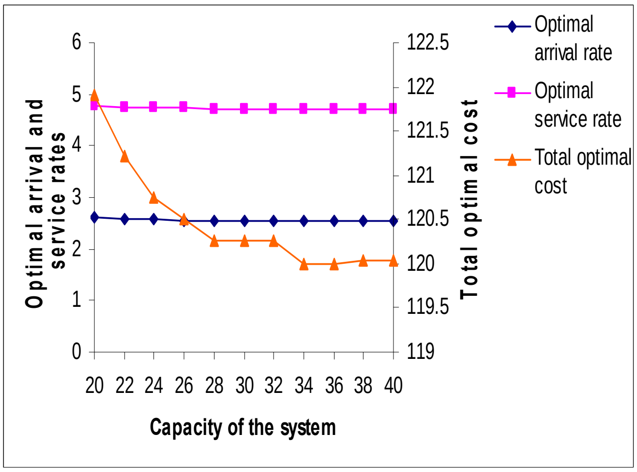

Fig.5.1: Capacity of the System vs. Total Optimal Cost

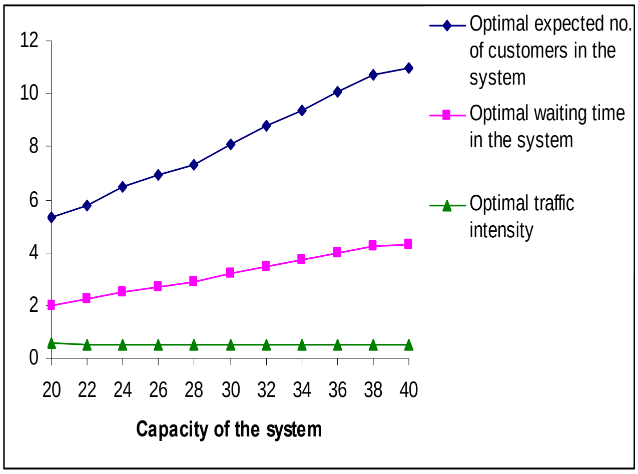

Fig.5.2: Capacity of the System vs. various Performance Measures

Observations: In Figure 1.1, we see that the optimum arrival and service rates do not vary as service cost increases which shows that the system is independent of service cost after a certain stage of arrival and service. As service cost increases the total optimal cost also increases. An increase of about 14.3% in service cost causes about 13.9% increase in total optimal cost. Hence these two costs show the positive correlation between them. In Figure 1.2, we observe that as service cost increases optimal expected number of customers in the system and optimal waiting time in the system also increase but the optimal traffic intensity remains constant. In fact 22.2% increase in service cost produces 14.8% increase in optimal expected number of customers in the system and 13.3% increase in optimal waiting time in the system. Thus these two performance measures are in positive correlation with service cost.

In Figure 2.1, we observe that the optimum arrival and service rates do not vary as waiting cost increases which shows that the system is independent of waiting cost after a certain stage of arrival and service. As waiting cost increases the total optimal cost also increases but quite slowly. In Figure 2.2, we see that as waiting cost increases the optimal expected number of customers in the system and optimal waiting time in system decrease quite slowly and traffic intensity does not vary significantly. It is almost constant. Thus there is a weak correlation between waiting cost and optimal expected number of customers in the system and optimal waiting time in the system.

In Figure 3.1, we see that as the number of servers increases the optimum arrival and service rates are also shown as fluctuating. As number of servers increases the total optimal cost fluctuates but it shows an increasing trend. In this way, a positive correlation between number of servers and total optimal cost can be seen here. In Figure 3.2, we observe that optimal expected number of customers in the system fluctuates in the beginning and then shows decreasing trend whereas optimal waiting time in the system also fluctuates in the beginning but shows increasing trend as number of servers increases. The optimal traffic intensity of the system decreases as number of servers increases. In fact 28.5% decrease is observed in optimal traffic intensity by increase of one server.

In Figure 4.1, we find that the number of customers in the system *m* increases but optimum arrival and service rates do not vary significantly. As *m* increases the total optimal cost increases. In fact, an increase of one customer in the system causes about 9.8% increase in total optimal cost. In Figure 4.2, we observe that optimal expected number of customers in the system, optimal waiting time in the system, and optimal traffic intensity does not vary significantly as number of customers in the system *m* increases.

In Figure 5.1, it may be of interest to note that the optimum arrival and service rates are not varying as capacity of the system increases which shows the stability of the system. Moreover, there is no correlation between capacity of the system, optimum arrival and service rates. As capacity of the system increases the total optimal cost does not vary. In Figure 5.2, we observe that optimal expected number of customers in the system and optimal waiting time in the system increase as capacity of the system increases and increase of two units in the capacity of the system causes 9.4% increase in optimal expected number of customers in the system and 10.8% increase in optimal waiting time in the system. Thus both the performance measures of the system are in positive correlation with the capacity of the system. The optimal traffic intensity does not vary with capacity of the system.

5. CONCLUSION

Finally, we conclude with the remark that present research on the computation of optimal performance measures of the truncated multi-channel queuing model with Poisson arrival, exponential service, and finite capacity can pave the way for future progress of research in various fields including technical applications for the digital communication systems and as well as in assessing the performance measures in the form of optimal arrival, optimal service, and optimal cost of computer networking by applying this queuing approach. The aim of the numerical demonstration is to study the variability of the model that is, to assess the effect of one parameter on the others especially such parameters which characterize the performance measures of the model. Numerical demonstration is carried out with the help of hypothetical data-input. The computer program developed in the paper can also be tested for any real case study at later stage. It has also a good deal of potential to the applications in other areas such as inventory management, production management, computer system etc.

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